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GETTING SMART ABOUT ASSESSMENT FOR LEARNING

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"Specific Mathematics Assessments that Reveal Thinking"—or smart tests—provide teachers with a quick and easy way to conduct assessment for learning. Using the internet, students in years 7, 8, and 9 undertake a short test that is focussed strongly on a topic selected by their teacher. Students' stages of development are diagnosed, and sent to the teacher within minutes. Many tests have been produced and are now being trialled in 7 Victorian schools. Where available, on-line teaching resources are linked to each diagnosis, to guide teachers in moving students to the next stage. This project is sponsored by the Australian Research Council and Victoria's Department of Education and Early Childhood Development.

Introduction

A Year 9 class has just had a lesson introducing trigonometry. All seems to be going well … or is it? The teacher has the students do a quick on-line test and within minutes receives feedback revealing that four of the students cannot identify sides in relation to given angles. She spends a few minutes with this group doing some targeted teaching, and they are then able to catch up with the rest of the class.

Teaching mathematics is a wonderful career. The subject area is fascinating and useful. Society values people with mathematical skills and there are incentives for students to do well. But there are challenges. In every secondary mathematics classroom, there is a huge spread of ability and mathematical knowledge. It has been said (Hart, 1981) that in a truly mixed ability class of early high school students there is a seven-year range of achievement between the strongest and the weakest student. Teachers work with classes where some of the most able students are having extra mathematics lessons so in some topics are well ahead of their classmates, whilst other students have substantial difficulty and little enthusiasm to improve the situation. These are the dilemmas of many of us who are trying to do the very best that we can for each of our students.

Understanding new mathematical concepts often relies on having good background knowledge and so, to avoid presenting some of the class with tasks that they cannot do, we sometimes excessively revise earlier material and make sure that the tasks set for the bulk of the lesson are straightforward enough to be tackled by anyone. It is often difficult to keep track of students' progress and identify exactly where they are struggling. It is often hard to identify when students are ready for the current topic, and exactly where they might be having difficulty.

It is with all of this in mind that a partnership was set up between the University of Melbourne, the Department of Education and Early Childhood Development, and 7 state secondary colleges; Buckley Park, Gladstone Park, Melbourne Girls', Taylors Lakes, Ashwood, Bayside and Princes Hill.

The brief was to develop a way of making assessment for learning a practical tool in junior secondary classes. We set about designing "smart tests" (HREF1) that could give teachers information about the understanding of their individual students in key mathematics topics. A "smart test" is a "**s**pecific **m**athematics **a**ssessment that **r**eveals **t**hinking" Most commonly, they focus on fundamental understanding of essential ideas, although as illustrated in the anecdote at the beginning of the paper, some assessments also target simple skills. Feedback to teachers includes the diagnosis of many of the common misconceptions and suggestions as to how the elimination of these misconceptions could be approached. These smart tests were to supplement the excellent assessments that Victorian teachers have developed and used over a many years: they are focused narrowly on precise topics to maximize relevance to teaching, and do not give an overall level of performance.

Now, at the end of 2009, we have an extensive set of online quizzes that has been developed to inform classroom teachers about the understanding of students in their classes. Teachers read descriptions of the available smart tests, choose one that is appropriate, and give students a password to do it. The students' attempts are marked by computer and the patterns of results are electronically analysed. Each student's results, along with information on the common misconceptions in the topic and relevant links to the Victorian *Mathematics Developmental Continuum P-10* (HREF2) to address the issues raised, are available as soon as the teacher logs in. In the following sections we describe the educational and design philosophy behind the smart tests and illustrate this with an example. Further information on the project and the smart tests is available on the project website (HREF1).

Smart tests — Educational and design philosophy

A smart test focuses on a single important concept. Our previous work with the *Continuum* (HREF2) had highlighted the importance of students needing to understand certain critical concepts before being able to progress. Our aim with the smart tests is to target some of these critical concepts, and design

- short and easy to administer on-line diagnostic assessment;
- prompt feedback to teachers about class and individual performance;
- targeted teaching suggestions that address the conceptual hurdle.

These components, together, highlight the purpose of the smart tests as "assessment for learning". Smart tests are not designed to give a score, but identify the stage of understanding that the student has reached, diagnose misconceptions and provide teachers with information that will help them meet students' needs and improve learning outcomes.

There are several types of smart tests. The most important assess underlying mathematical conceptual understanding, and would generally be used before beginning to teach a topic that builds on these ideas. An example is given below. Some other smart tests check students' knowledge of facts and skills to report to the teacher whether prerequisite understanding is in place prior to teaching a new topic. Although these do not have the conceptual emphasis of the other smart tests, these were created because we know that missing background knowledge can significantly hinder students' progress. Both types of tests help teachers to target their teaching to individual students' needs. Most tests come in matched pairs, which can be used as pre-test and post-test, so teachers can track students' progress.

An example: Multiplication by numbers less than one

To illustrate the purpose, design, and components of a typical smart test, we present an item from a smart test that identifies misconceptions involving multiplication and division. We shall describe the background educational issues and present the item, and then show the diagnosis that is provided to teachers after students have completed the test, along with a discussion of the kinds of teaching suggestions that are made.

The concept and the associated smart test item

One of the well-known misconceptions in the area of number operations is that "multiplication makes bigger, and division makes smaller" (MMBDMS) (Bell, Swan, & Taylor, 1981). As with many misconceptions, MMBDMS arises as a natural consequence of previous learning. When students first learn about multiplication and division, it is with whole numbers, and multiplication does indeed generally make bigger (e.g. $2 \times 5 = 10$ and 10 is greater than both 2 and 5), and division does, indeed, generally make smaller. Strong foundational learning like this is essential for students' progress, but to go further in mathematics, students have to simultaneously build on these concepts and learn how they work in new situations. In the world of fractions and decimals, when multiplying by numbers less than one, the formerly useful whole number principle of MMBDMS becomes a misconception. Fortunately, like many other misconceptions, MMBDMS can be readily addressed. Left unaddressed, it can remain to plague students throughout their schooling.

One of the other issues associated with number operations is whether or not students can choose the correct operation when faced with a word problem. A person's ability to solve a word problem is dependent upon:

- Step 1: Recognizing the structure of the problem and hence choosing the appropriate operation(s), and
- Step 2: Performing the calculation(s).

While a calculator can be used to assist with Step 2, it will not provide a correct answer if appropriate operations are not selected in Step 1. Choosing the right operations for solving word problems is even more fundamentally important than being able to calculate.

These issues—the MMBDMS misconception and choosing the right operation in a word problem—are addressed in the smart test, of which one three-part item is shown in Figure 1. Notice that this item does not require students to do any calculation at all, but merely to select from three choices in a drop-down menu. The drop-down menu for the mussels problem is shown in the figure and involves choices among multiplication and division operations; the flake and flathead problems have similar multiple choices.

As can be seen, all of the problems in Figure 1 are about multiplication, but the nature of the numbers varies. In the past, when computation was done by pencil and paper, it was thought that students did particularly badly on the third problem because of errors of calculation. When calculators became available for student use in the late 1970s, researchers expected that the difference in the difficulty of the three problems would go away because the process of multiplication was done by the calculator (Stacey, 2009) and so nearly all students should get *all* these problems correct. To their surprise, problems involving the first two parts become easy, but even with calculators the third remains difficult. This is *not* because of reading difficulties (reading problem 3 is no harder than reading the others) but rather because the MMBDMS misconception makes students pick the wrong operation. When we ask them to *estimate* the answers, students can give good answers to all three problems (e.g. the flathead costs about $26 and the mussels cost about $10). They *do* understand the problem and the real situation, but they still cannot actually solve the problem with the given numbers, with or without a calculator. The reason is that they know the answer for the mussels should be smaller than the $13.40 that they cost per kilogram, and they pick division (not multiplication) because they think that division must be involved in order to get this smaller answer.

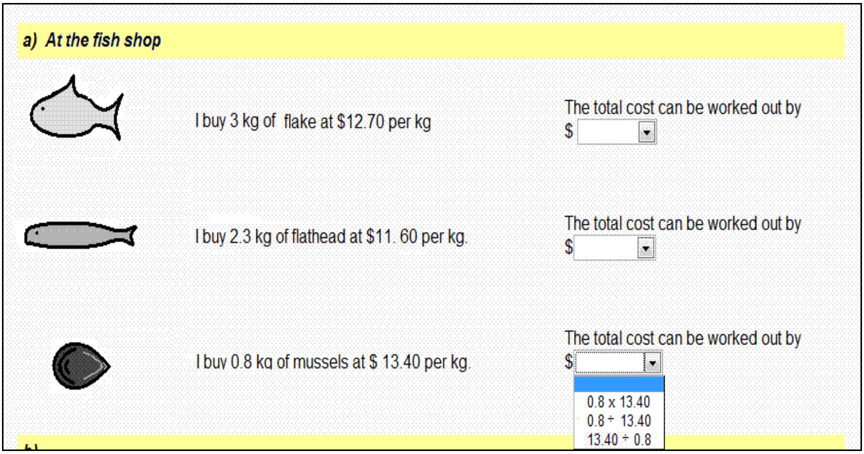

Figure 1. The fish shop item probes misconceptions
with multiplication and division.

Students doing the smart test would complete the item in Figure 1, together with some additional items that make it possible to diagnose, with reasonable confidence, whether or not they have any misconceptions. The idea is that these tests are simple and quick for students to complete, and that it is concepts being targeted rather than the ability to compute. Indeed, care has been taken to reduce having computational issues interfere with diagnosis of students' conceptual understandings as they undertake the tests. In some smart tests an electronic calculator is available.

The diagnosis

As soon as the students submit their responses online, the results are analysed electronically using carefully designed algorithms that recognizes patterns of responses corresponding to different types of typical thinking, and based on mathematics education research results from around the world. The results allow teachers to identify any global problems, or to group students for targeted teaching.

Developmental Stages and Teaching Suggestions

Choosing multiplication and division operations

- Mathematical focus and overview of developmental stages
- Detailed explanations of stages and common misconceptions
- Teaching suggestions
- References

Mathematical focus and overview of developmental stages

This module tests whether students can identify the correct operation to use in simple word problems involving multiplication and division, such as:
(a) What is the cost of 3 kg of fish at $18.50 per kg?
(b) What is the cost of 2.4 kg of fish at $18.50 per kg?
(c) What is the cost of 0.4 kg of fish at $18.50 per kg?
The module identifies the misconception that "multiplication makes bigger and division makes smaller", which causes students to be correct on questions (a) and (b) above, but to select division instead of multiplication to solve (c). Students with this misconception (those at Stages 2 and 3) are unable to solve these word problems correctly, even with a calculator.

Stage 1	Students can correctly identify the operations of multiplication and division in simple word problems when the multiplier or divisor is a whole number,
Stage 2	and also when the multiplier or divisor is a decimal or fraction greater than one,
Stage 3	and also, for Stage 3a when the multiplier or divisor is a decimal (but not a fraction) between 0 and 1 OR for Stage 3b, when it is a fraction (but not a decimal) between 0 and 1,
Stage 4	even when the multiplieror divisor is between 0 and 1.

One further misconception is flagged:

Misconception	Student is consistently using a ÷ b where b ÷ a is required.

Figure 2. Diagnostic feedback for teachers from problems shown in Figure 1.

The diagnosis includes information for teachers about the different stages of understanding that are revealed by the smart test. An example is shown in Figure 2. Each student is classified by their stage of understanding. In some cases there may be a few students who don't fit any pattern and so are not classified, and if this occurs it is included in the feedback as well. Teachers can interview these students to establish their real level of understanding. In this example, the diagnosis is reported in 5 stages (0 to 4). These stages relate only to this topic – they are not yet linked to an external framework such as VELS levels.

Teaching strategies

Included with the diagnostic information is a set of ideas for teaching, often differentiated by stage. In the case of the example in Figure 1, the advice for Stage 0 and Stage 1 students is to work on recognising the structure of word problems that involve multiplication or division. It is likely that many stage 1 students use repeated addition instead of multiplication. This strategy fails at the second problem, when the multiplier is not a whole number. So for them, they need to recognise problem situations where multiplication and division are appropriate, such as equal groups and rates. Relevant progression points and sections in the Mathematics Developmental Continuum are:

- 2.25 Early division ideas
- 2.75 Multiplication from equal groups to arrays
- 3.25 Choosing multiplication and division

Stage 2 and Stage 3 students have the MMBDMS misconception. Like Stage 0 or 1 students, they may also need to strengthen their recognition of situations (equal groups, rates etc) that involve multiplication and division. However, they have moved on from repeated addition. They need to learn that the type of number in the problem does not change the operation and to learn to estimate the effect of multiplying and dividing by numbers less than one (e.g. to estimate that 0.4 x 34.5 is just under half of 34.5, so about 16.)

For ideas on developing concepts of multiplying and dividing by numbers less than 1, see the following indicators in the Mathematics Developmental Continuum:

- Conceptual obstacles when multiplying and dividing by numbers less than 1 (level 5.0)
- Number: The meaning of multiplication

As with all contents of the Continuum, these individual items can be readily located by using the "search" function on the site.

Using the smart tests

Each smart test is completed on-line and only takes a few minutes. As mentioned earlier, the tests involve only minimal calculation; there is also very little typing required. Responses are given by choosing from options in drop-down menus, selecting a radio button, dragging and dropping, and typing short numerical entries. There are many tests now available for key topics in the Years 7, 8 and 9 curricula, and further smart tests continue to be developed. Among the topics currently addressed are algebra, measurement (including area and perimeter), preparation for Pythagoras and trigonometry, basic understanding of decimals and fractions, and statistics and probability.

Teachers can choose how best to use the smart tests in order to suit their needs and facilities. The whole class could do a test simultaneously if there are computers available for everyone, students might do the test a few at a time, students might complete a test for homework, or a teacher might ask just a few students to complete a test because misconceptions are suspected among the group. The diagnostic information and teaching suggestions are available within a few minutes of students completing the smart test.

Conclusion

The usability of the smart tests and the opportunities they provide for quick, targeted diagnosis sound wonderful, although the task is not yet complete. The potential is definitely there, but the collection of smart tests is still a work in progress. There is some hard work ahead to ensure that this becomes a useful resource available to all teachers of mathematics, with a wide range of tests, accurate and reliable diagnoses based on the internationally most authoritative research on student development, and helpful teaching suggestions. We are actually ahead of schedule in some respects because we were able to start implementing the on-line aspect of the project right from the beginning, although as seen earlier this has provided challenges.

In the future we will continue to refine the power of the diagnostic algorithms and extend the teaching suggestions, as well as researching good ways to use the tests, and the effect on teachers' pedagogical content knowledge and students' learning. Currently the feedback goes only to teachers, and we are looking at ways of providing feedback to students as well, again as soon as they have completed the test. The challenge here is that students, too, need to understand that the tests are about diagnosis rather than giving a score.

We are optimistic, however, that the smart tests will be a powerful resource for diagnosing students' thinking, easy for schools to use, informative for teachers, and thus an important component of the assessment for learning process.

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