Journal of the Association of Teachers of Mathematics

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Mathematics Teaching

Issue 235
July 2013

FOCUS ON: DIGITAL TECHNOLOGY

www.atm.org.uk

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Mathematics Teaching

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Aims of ATM

The Association of Teachers of Mathematics aims to support the teaching and learning of mathematics by:

- encouraging increased understanding and enjoyment of mathematics
- encouraging increased understanding of how people learn mathematics
- encouraging the sharing and evaluation of teaching and learning strategies and practices
- promoting the exploration of new ideas and possibilities
- initiating and contributing to discussion of and developments in mathematics education at all levels

Guiding Principles

The ability to operate mathematically is an aspect of human functioning which is as universal as language itself. Attention needs constantly to be drawn to this fact. Any possibility of intimidating with mathematical expertise is to be avoided.

The power to learn rests with the learner. Teaching has a subordinate role. The teacher has a duty to seek out ways to engage the power of the learner.

It is important to examine critically approaches to teaching and to explore new possibilities, whether deriving from research, from technological developments or from the imaginative and insightful ideas of others.

Teaching and learning are cooperative activities.

Encouraging a questioning approach and giving due attention to the ideas of others are attitudes to be encouraged. Influence is best sought by building networks of contacts in professional circles.

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GC Reports PAGE 5

Further pure mathematics with technology: Developing a new A-Level mathematics unit that uses technology in the teaching, the learning, and the assessment - Tom Button PAGE 6

Why has technology failed to become commonplace in the A-Level mathematics classroom? Is it that, in terms of the hardware and the software, there are too many options on offer and too many choices to be made? Maybe expense is cited as a significant reason? Or is it simply that the use of technology is not a requirement of the assessment and final examination? Here, by including technology in the examination format, the use of technology to support the teaching and learning of mathematics is ensured. First assessment is in 2013 ...is this the shape of things to come at A-level?

Smart online assessments for teaching mathematics - Beth Price, Kaye Stacey, Vicki Steinle, and Eugene Gvozdenko PAGE 10

Assessment is 'part and parcel' of mathematics teaching. Teachers make assessments as they interact with students in many ways and in a variety of formats and media. The best assessment has a diagnostic element within its functionality. It is important for assessment to be reported upon within the shortest possible time-frame. Technology can enable sophisticated assessment to be engaged with and reported upon swiftly, and in the classroom. Here is an example of sophisticated online assessment that is available to all teachers... now. Just reach for your tablet, or laptop.

Working with European colleagues to develop technology in mathematics classrooms - The EdUmatics project - Cindy Hunt PAGE 17

Mathematics education is a global reality, but unfortunately the opportunities to engage with colleagues working in different cultures are few and far between. EdUmatics was funded by the EU, and set out to involve teachers and mathematics educators in many countries. Technology is not a futuristic notion, technology is now the reality, yet many mathematics classrooms and much mathematics teaching exhibit little or no evidence of application of appropriate technologies, nor is there an appreciation of the 'powerful' ways in which technology can support learning. Will it always be thus?

Mathspen: Envisioning the future - Mandy Lo PAGE 20

Notation in mathematics is a double-edged tool. It enables complex mathematical ideas to be communicated unambiguously throughout different cultures and geographical regions of the world. Yet this very notation is a significant disincentive when it comes to interactive dialogue using virtual worlds. Will MathPen be the game-changer when it comes to communicating with mathematical notation?

Using Derek Ball's Triangle Problem to explore mathematical processes, and taking time to discover the obvious - Jo Tomalin PAGE 23

Problems that are often 'simply stated' can present surprising challenges. Problem solving is a 'personal' process that begins with 'finding somewhere to stand'. Thereafter the journey to solution is determined by decisions made 'along the way'. This account displays the step-by-step approach taken by those engaging with the problem and, on occasions, a commentary that might be regarded as self-assessment.

QR codes in the mathematics classroom - Gill Leahy PAGE 27

QR codes are not a new phenomenon they have been seen and used in publications for some time – even MT. The frequency of use will depend on the specific interest of the reader, the technological sophistication of the reader, and ready access to a hand-held device with an appropriate QR scanner/reader installed. A QR code can be a direct link from the page, leaflet, poster, container, advertisement, etc. to a virtual environment. Some might say that QR codes are the route to the most accessible interactive learning experience for many learners. The mobile 'phone is an everyday part of life for students today. Student 'phones are available, familiar, charged, and offer more uses than can be appreciated, or comprehended by most users. So, embrace the technology and turn the 'phone/handheld device into a powerful classroom tool.

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'I get it now!' Stimulating insights about probability through talk and technology - Sibel Kazak, Rupert Wegerif, and Taro Fujita PAGE 30

Simulation is a 'gift' to technological approaches. Vast numbers of 'events' can be simulated and data recorded 'at the push of a button'. Data sets can be real, can relate to the parameters set by the user, and can be used to 'make sense' of a situation and enable understanding of a mathematical concept or process. Here, is an explanation of how *TinkerPlots* can support the learning and teaching of probability annotated with comments from the learners 'as it happened'.

Postscript - Bob Burn PAGE 33

Learning to use new technologies - embrace those lesson hiccups! - Alison Clark-Wilson PAGE 34

The mathematics classroom is a dynamic environment where the unexpected is normal. Coping with the unexpected is what mathematics teachers do, seemingly by deploying strategies from their established classroom toolkit. However add technologies to the classroom mix and suddenly things are very different. This is because the unexpected associated with new technologies is also unanticipated through lack of experience. This combination is a feature of all professional development, but when technology is involved somehow the anti is more than just 'upped'!

The role of the teacher in groupwork - Chris Stephens and Ros Hyde PAGE 37

Working in groups in mathematics is the norm in primary schools, yet beyond Year 6 working in this way seems to be something of an event. There are many reasons for this, but clearly different skill sets are needed, by both teacher and learner if working in groups is to be effective. Teachers need to manage the process from task selection through to determining appropriate learning groups. Students will work collaboratively, discuss, support and challenge each other if the learning environment, generated by the teacher, is good. So, that is the what? sorted, now for the how? Here classroom events are described and contextualised, task selection, preparation, group structures, and the importance of listening are all discussed. Students learning from each other is a powerful strategy, but engineering the process is not a single strategy approach for the teacher.

Hands on = Brains on – David Murrells PAGE 40

Not only is this a good read, it offers a template for the use of technology that can signpost both thinking and application to future practice for the reader. In the words of the author... *One of the most important things that I have realised is that one needs to invest time and energy in allowing students to get to grips with the software before they can begin to use it as a mathematical tool. I see this as a transition from pushing physical buttons to pushing mental buttons.*

Embedding dynamic technologies in the key stage 3 curriculum – The Cornerstone Maths approach - Liz Gould, Samuel Ikhinmwin, Alex Walley, Alison Clark-Wilson and Celia Hoyles PAGE 44

Here is an example of what might be achieved if creative mathematical approaches are linked to the power and potential that technologies can offer. The ideas are developed and tried in real classrooms that exhibit all the usual inhibitors to learning. The project still has work to do, but the fundamentals that underpin the process are likely to ensure that the outcomes will enhance learning in many classrooms. Not only that, you are invited to become involved. Can you afford not to be part of the team?

***Autograph* in the classroom - Alan Catley** PAGE 48

Optimising the use of software to support teaching and learning relies on a familiarity with the functionality that develops over time with 'use'. Here some of the 'tools' available are both explained and exemplified. The potential of the software is clear, and the explanations are well considered. There is even an offer of resources developed for use with *Autograph*. As the author says *Autograph* is *'easy to use'*.

Maths Medicine 6 - Dietmar Küchemann PAGE 51

A problem for the reader and students to work on. There is a link to the web-site where suggestions for use and a solution can be found alongside interactive files.

Editor Margaret Jones
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Alison Clark-Wilson

SMART ONLINE ASSESSMENTS FOR TEACHING MATHEMATICS

Beth Price, Kaye Stacey, Vicki Steinle, and Eugene Gvozdenko outline the rationale, design and potential impact of an online resource

Introduction

A Year 9 class has just had a lesson introducing trigonometry. All seems to be going well... or is it? The teacher asks students to do a quick online test and within minutes receives feedback revealing that four of the students cannot identify sides as adjacent, or opposite, in relation to given angles. She spends a few minutes with this group conducting some targeted teaching, and they are then able to catch up with the rest of the class. This group is then asked to complete short tests on comparing ratios and calculating with proportions to find out if they have a strong conceptual understanding of these topics, which are intimately connected to trigonometry.

Teaching mathematics is a wonderful career. The subject area is fascinating and useful. Society values people with mathematical skills and there are incentives for students to do well. But, there are challenges. Every secondary mathematics classroom contains students with a range of attitudes, experiences, understanding and skills. Over 30 years ago, Kath Hart (1981) reported that, in a typical class of early high school students, there was *a seven-year range of achievement*. There has been other research which has confirmed this wide range. Teachers work with classes where some of the high-achieving students are well ahead of their classmates, whilst other students have substantial difficulty and little enthusiasm to improve the situation. These are the dilemmas that many of us face, and we are trying to do the very best that we can for each of our students.

Understanding new mathematical concepts often relies on having a good background knowledge and so, to avoid presenting some of the class with tasks that they cannot do, we sometimes excessively revise earlier material. We also make sure that the tasks set for most of the lesson are straightforward enough to be tackled by all students, hence limiting the more challenging material. It can be time-consuming to keep track of students' progress and difficult to identify exactly where some students are struggling as well as the reasons for this.

It is with all of this in mind that the smart test system was designed. The brief was to develop a way of making "*assessment for learning*" a practical tool for teachers of students in Years 7, 8 and 9 (Stacey, Price, Steinle, Chick & Gvozdenko, 2009). This has since been expanded to include Years 5 and 6 as well. We set about designing "*smart tests*" that could give teachers information about the understanding of their individual students in key mathematics topics. A "*smart test*" is a "*specific mathematics assessment that reveals thinking*" Most commonly, smart tests focus on fundamental understanding of essential ideas, like ratio and proportion. However, as illustrated in the anecdote at the beginning of the paper, some assessments also target prerequisite skills - identifying adjacent and opposite sides, for example. Feedback to teachers consists of a description of each student's '*developmental stage*' and the diagnosis of many of the common misconceptions. Teaching suggestions are provided which are intended to assist with the removal of these misconceptions and to move students on to the next developmental stage. Smart tests were designed to supplement, rather than replace, the excellent assessments using rich tasks that teachers have developed and used over many years. Smart tests are focused narrowly on precise topics to maximize relevance to teaching, and do not give an overall level of performance. The developmental stages reported are topic-specific, and not systematically linked to any overall system of development or government achievement targets.

We now have an extensive set of online tests that has been developed to inform classroom teachers about the understanding of students in their classes. These cover about 60 topics, with two versions of most tests. Teachers read descriptions of the available smart tests, choose one that is appropriate, and give students access via a code. The computer marks the students' responses and the response patterns are automatically analysed to diagnose the student's developmental stage in the topic as well as any misconceptions or common errors.

Each student's diagnosis is available to the teacher as soon as he or she accesses the teacher page of the smart test system.

In the following sections we describe the educational and design philosophy behind the smart test system and illustrate this with an example. Further information about the project is given in Stacey, Price, Steinle, Chick, Gvozdenko, (2009). Access to the 'smart test' system is available on the project website.

The smart test system - Educational and design philosophy

A smart test focuses on a single important concept or skill. The previous work of our team members and many other researchers had highlighted the importance of students needing to understand certain critical concepts in order to make solid progress in new topics. Our aim with the smart tests is to target some of these critical concepts, and design features;

- short, and easy to administer, online diagnostic assessment;
- immediate feedback to teachers about class and individual performance;
- targeted teaching suggestions that address the conceptual hurdle.

These components, together, highlight the purpose of the smart tests as "*assessment for learning*". Smart tests are not designed to give a score, but to identify what the student knows, to diagnose misconceptions, to provide teachers with information that will help them meet students' needs, and to improve learning outcomes. To do the diagnosis, complicated algorithms have been designed to look for tell-tale patterns of systematic errors in student responses.

There are several types of smart tests. Most assess underlying mathematical conceptual understanding, and would generally be used before beginning to teach a topic that builds on these ideas. An example is given later. Some other smart tests check students' knowledge of facts and skills to report to the teacher whether prerequisite understanding is in place prior to teaching a new topic. An example is "*Labelling hypotenuse, opposite, adjacent sides for trigonometry*" discussed above. Although these do not have the conceptual emphasis of the other smart tests, they were created because we know that missing background knowledge can significantly hinder student progress. Both types of tests help teachers to target their teaching to individual students' needs. Most tests come in matched pairs, which can be used as pre-test and post-test, so teachers can track student progress.

An example: Readiness for operations with directed number

To illustrate the purpose, design, and components of a typical smart test, we present an item from a smart test that identifies both developmental stages and misconceptions involving students' understanding of integers. We will describe the background educational issues and present the item, and then show the diagnosis that is provided to teachers after students have completed the test, along with a discussion of the kinds of teaching suggestions that are made. To view this and other smart tests, follow the instructions on the smart test home page. http://www.smartvic.com/

The concept and an associated smart test item

Many teachers will be aware that for some students, operations with directed numbers remain a difficulty throughout their study of mathematics. We think this difficulty has two sources. The first lies with the general concept of directed numbers. They are not numbers used to count normal objects, but numbers which describe numerical information with respect to a reference point: how many, or how far above or below a baseline quantity, to the left or the right of a position, or to the north or the south of a location, for example. To understand operations with negative numbers, initially addition and subtraction and multiplication as repeated addition, students need to be able to interpret operations as movements up and down a number line. The basic ability to move up and down the number line is the second aspect tested

The intention is that the smart test "*Readiness for directed number*" be used before secondary school teachers teach formally about operations on directed numbers. We have used the task of ordering positive and negative integers as a quick indicator of the underlying conceptual understanding. One such item is shown in Figure 1 (See page 12). The item requires students to drag the number cards into the correct order. The number cards snap into place. The precise choice of the numbers in this item is important, to enable us to identify relevant misconceptions. We have used several items to test students' ability to move up and down a number line, in preparation for operations. Students move the mercury level up and down on virtual thermometers to show temperature increases and decreases. In these items, the visual labelled temperature scale is provided so that students can directly count the number of steps that are being taken. Other items about continuing number patterns across zero have no such visual support. Because we know that students have other difficulties with non-integer (decimal) negative numbers, there is a separate test that reports on that topic.

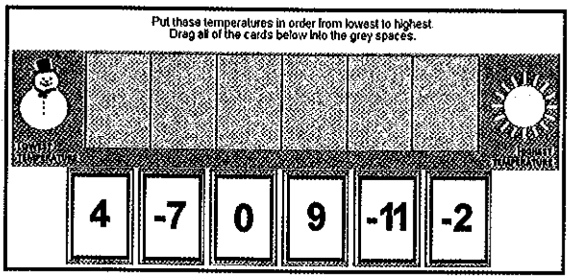
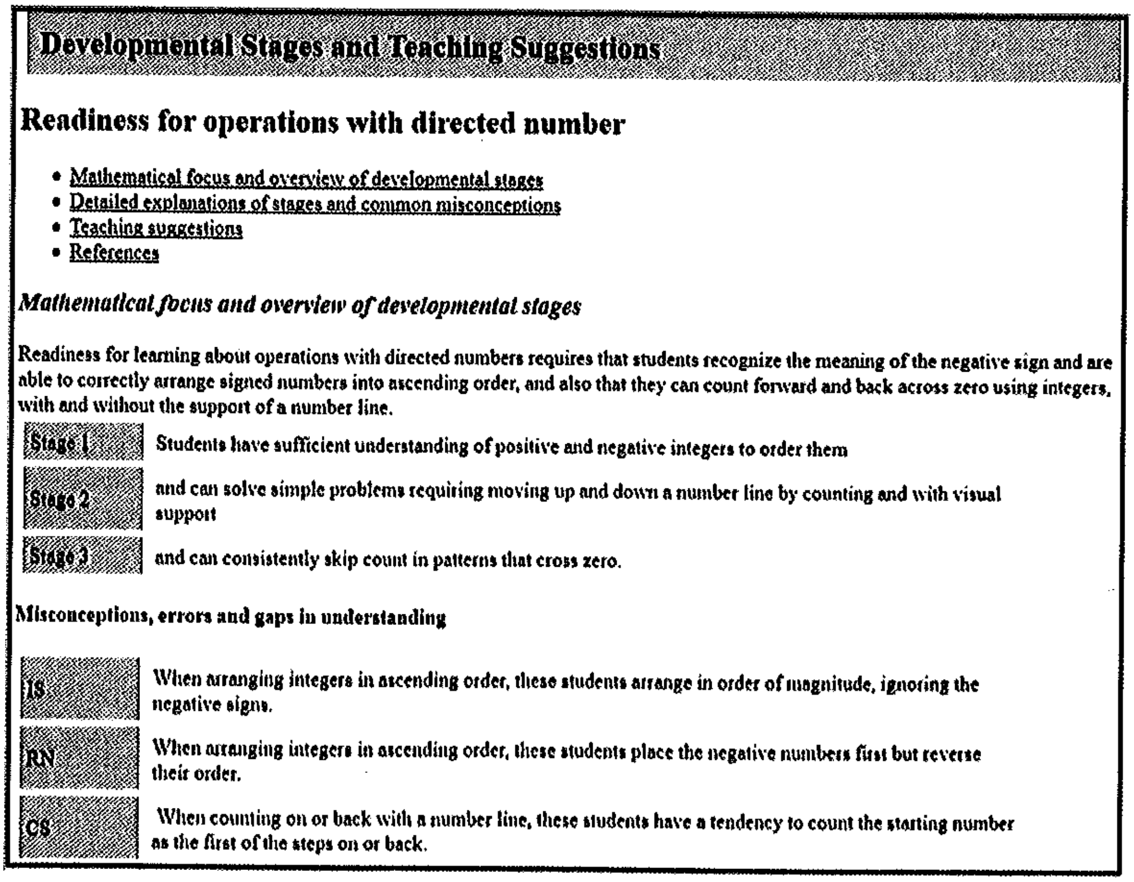


Figure 1. The first "Readiness for operations with directed number (test B)" item probes understanding of the order of negative integers and identifies some misconceptions.

Students doing this smart test complete the items, starting with that in Figure 1. By looking at the exact responses and the patterns in all their responses, it is possible to diagnose, with reasonable confidence, what students can do, and whether or not they have any misconceptions. The idea is that these tests are simple and quick for students to complete, and that it is concepts being targeted rather than the ability to compute. Indeed, care has been taken to reduce having computational issues interfere with diagnosis of students' understandings as they undertake the tests. In some smart tests an electronic calculator is available. However, we find that the general format and students' expectations of computer testing mitigates against items which require much '*working out*': in general, students are reluctant to go to pen-and-paper to complete an item on the screen, so even with a calculator available, multi-step items are not suitable. This is not a great restriction for items targeting conceptual understanding. However, it does highlight the need for teachers to conduct their own assessments, including multi-step items.



Developmental Stages and Teaching Suggestions

Readiness for operations with directed number

- Mathematical focus and overview of developmental stages
- Detailed explanations of stages and common misconceptions
- Teaching suggestions
- References

Mathematical focus and overview of developmental stages

Readiness for learning about operations with directed numbers requires that students recognize the meaning of the negative sign and are able to correctly arrange signed numbers into ascending order, and also that they can count forward and back across zero using integers, with and without the support of a number line.

Stage 1	Students have sufficient understanding of positive and negative integers to order them
Stage 2	and can solve simple problems requiring moving up and down a number line by counting and with visual support
Stage 3	and can consistently skip count in patterns that cross zero.

Misconceptions, errors and gaps in understanding

IS	When arranging integers in ascending order, these students arrange in order of magnitude, ignoring the negative signs.
RN	When arranging integers in ascending order, these students place the negative numbers first but reverse their order.
CS	When counting on or back with a number line, these students have a tendency to count the starting number as the first of the steps on or back.

Figure 2. Diagnostic feedback for teachers from "Readiness for directed number" smart test.

Because the assessment is automated, smart tests require responses that are machine-interpretable. The range of these response types is growing rapidly (Stacey & Wiliam, 2013). As well as single closed numerical answer and traditional multiple choice, where we provide alternatives derived from research results as far as possible, there are *drag-and-drop* items of many varieties, and the use of sliders enables students to place numbers on number lines, make bar graphs etc. We hope some students at least will find the items bright, interesting and engaging through their different activity formats.

The diagnosis

As soon as students submit their responses online, the results are analysed electronically using carefully designed algorithms that recognize patterns of responses corresponding to different types of typical thinking. The diagnosis includes information for teachers about the different developmental stages that are revealed by the smart test. The developmental stages are based on mathematics education research results from around the world, wherever possible, (e.g. Küchemann, 1981) but moderated by an analysis of student results on the smart tests, and by what we hope teachers will find practical and useful. Stacey, Price and Steinle (2012) explains this process. The results allow teachers to identify any global problems in the whole class, or to group students for targeted teaching, and also to move students on more quickly if they show good understanding.

The developmental stages and misconceptions information for the "*Readiness for directed number*" smart test are shown in Figure 2 (See page 12). Each student is allocated to a developmental stage. In some cases the pattern of incorrect responses shows a particular misconception, and when this occurs it is also included in the feedback for the student. In this example, the diagnosis is reported in 4 stages (0 to 3) and any of 3 misconceptions will be reported when they occur. These stages relate only to this topic – they are not linked to an external framework, although we believe they will relate well to most curriculum documents. Another example is described in Price, Stacey, Chick and Gvozdenko, (2009).

At the beginning of 2013, this smart test was used by 163 Year 8 students of volunteer teachers. In this sample, 49% had reached Stage 3 and so were ready for formal work on operations. They can order integers, and move up and down a number line whether it is shown or not. There were 8% of students at Stage 2. These students could move up and down the number line (e.g. thermometer) when it was shown, so that students could answer correctly by counting along the scale by ones instead of calculating. The 29% of students at Stage 1 were able to place the number cards in order, but they were not able to move up and down the number line. They may have a generally vague conception of the number line, or they may have one of the misconceptions below. Finally 14% of this Year 8 sample was at Stage 0, i.e. below Stage 1, not yet ordering integers correctly, again possibly with identifiable misconceptions.

As can be seen in Figure 2, three misconceptions, or common errors, are reported by this test. In the sample of 163 Year 8 students, 2% had misconception IS, simply ignoring the negative sign in front of the numbers. This error would be more common in younger students. About 4% of students placed all the negative numbers on the left of zero, correctly, but ordered them as -2 < -7 < -11 (misconception RN). This misconception has been observed elsewhere. For example, Widjaja, Stacey & Steinle (2007) report it with decimal numbers. A further 12% of the students were observed to be systematically 'one out' when they moved up and down the number line (misconception CS), including when the number line is shown. For example, they say that 3 more than -2 is 0. Probably this is because they count 3 places as -2, -1, 0, counting the starting point, instead of -1, 0, 1, but it is also possible that some of them think there is a separate number -0, so that they count three places as -1, -0, 0.

A computer diagnosis from the smart test system gives good information, but not excellent information. For example, a student with good understanding might accidentally miss out some items and hence not jump the hurdles set in the programming for the early stages. We think this is a question of cost-effectiveness in educational design. If a teacher can get good information on whole classes with a small amount of effort, then they have time to investigate puzzling cases, for example through a short interview with the student 'live'. We know that there has been very good progress made through teachers interviewing all their students to establish their stages of learning see, Stewart, Wright & Gould, (1999). However, this is a very time consuming practice, with the information gleaned rapidly going out of date as children learn. Interviewing gives more information to the teacher, but it comes at a high cost.

To assist teachers in cases where results may be questionable, we make student responses on all items available to teachers who choose to investigate, flag students who omit many items, and flag students who have a high percentage of items correct overall but get a low stage. The latter students often have just made careless errors on easy items, but sometimes the test reveals fundamental misunderstanding.

Teaching strategies

Included with the diagnostic information is a set of ideas for teaching, generally differentiated by stage. In the case of *"Readiness for directed numbers"*, the advice for Stage 0 students is *to work on basic understanding of negative numbers*. As noted, these are not counting numbers, but give information about distance from a reference point and direction. These students need to use directed numbers to describe common situations, perhaps beginning with the straightforward example of temperature. They should describe a profit of £100 as +100, and a loss of £20 as -20, and order quantities within this model. Students should also use other common models such as distance north and south of a fixed point, as well as a number line. The symmetry of the magnitudes of numbers on a number line with zero at the centre should be stressed, and also that positive numbers are shown on the right and negative on the left. Students could be asked to order sets of integers, using situations such as temperature, or floors in a building above and below ground level. They could also be asked to write all the integers between, say, -6 and 3.

Stage 1 students have a basic understanding, but need to strengthen their mental image of directed numbers on a number line, temperature scale etc., and be sure about its details. The fact that 0 is not a signed number needs to be discussed with students, so that they appreciate that their intuitive ideas about -0 and +0 as two separate numbers are mistaken. Stage 1 students would benefit from exploring situations where they can count up and down a number line, such as finding the temperature after a change, or finding where a lift is, if it started at floor 3 and went down 5 floors.

Stage 1 students benefit from visual support, for example labelling their own number line or thermometer scale, and may well count by ones, but by Stage 2 students are becoming able to use a mental number line and to make elementary links with operations and number line movements. For example, to move on to Stage 3 and be ready to learn formal operations with negative numbers, the student has to reason in ways like this: "*this number pattern is subtract 3 each time – I do this by counting down 3 on my mental number line – in the positive region I subtract 3, but in the negative region it is like adding 3 (e.g. I go from -5 to -8)*". Stage 2 students will benefit from skip counting across zero. A simple activity is to use the constant addition or constant subtraction facility of simple calculators. The beginning of a pattern is provided, such as 8, 6, 4,... Students are asked to predict the next three numbers. Using a starting number of 8 and subtraction of the constant 2 students can see if their prediction is correct. Some 78% of the students studied were able to do this correctly. Of those who couldn't the major difficulty was with the third number, -2.

Stage 0 students may have one of the two identifiable ordering misconceptions, IS (*ignore negative sign*) and RN (*reverse order of negatives*). Like other Stage 0 students, they should work to understand negative numbers through the common models of integers. However, these students already have an incorrect idea in place. They need to be convinced that there is an error in their thinking before they realise that there is a need to do something different. Frequently students with misconceptions feel that they understand the situation and do not look for alternative ways of thinking about it. Similarly Stage 1 students may exhibit the CS (*count start*) misconception, which results in answers being one out – they count the (named) points on the number line, not the intervals for measurement. For example, a temperature change from -7 degrees to -2 degrees will be reported as 6 degrees (-7, -6, -5, -4, -3, -2). Going back to consider the real situations is useful. It is also important for students to develop awareness that this is an easy mistake to make and that good checking strategies are needed. For example, they can test their method on a simple problem: a temperature change from -6 degrees to -5 degrees is not 2 degrees.

Feedback from teachers

Many teachers feel that they have learned something that has been useful for them through using the smart tests (Steinle & Stacey, 2012). Teachers complete a voluntary survey after they access students' results from a test. It includes the multiple-choice question:

As a result of using this test, have you learned something useful for you as a teacher?

The results show that nearly all teachers who responded reported that they learned something useful and nearly half were very positive.

We had expected that many teachers would be alerted to students' inadequate preparation for learning a topic, and so have to start their teaching at a lower level than expected, and this was indeed the case. However, the opposite situation also occurred. Many teachers commented that they started their teaching of the topic at a higher level than they had previously planned after the diagnosis indicated that students had a better than expected understanding. As well as whole class teaching, many teachers reported using the diagnosis from the smart test system to form groups with similar needs. The most common strategy was to target specific difficulties by discussing misconceptions with groups of students who had the same diagnosis.

Accessing the smart test system

This year, 2013, access to the tests via the www.smartvic.com server (Stacey, Price, Steinle, Chick, Gvozdenko, 2009) is available to all teachers and their classes. Individual teachers can obtain access to the tests relevant to their classes, and are able to use them with their students. On the website, teachers can see test descriptions, choose three tests and try them out with students. After this, before access to a subsequent test is granted it will be necessary to complete a 10-minute online feedback questionnaire. In this fashion it is possible for a teacher to build up a set of tests to which they have access.

Each smart test is completed online and only takes a few minutes. As mentioned earlier, the tests involve only minimal calculation; there is also very little typing required. Responses are given by choosing from options in drop-down menus, dragging-and-dropping images, typing short numerical entries and moving sliders.

There are about sixty tests now available for key topics in the Years 5 to 9 curricula, and most of these have parallel tests available. Among the topics currently addressed are number, algebra, measurement, spatial visualisation, preparation for Pythagoras' theorem and trigonometry, basic understanding of decimals, fractions and percentages, and statistics and probability.

Teachers can choose how best to use the smart test system in order to suit their needs and facilities. The whole class could do a test simultaneously if there are computers available for everyone, students might do the test a few at a time, students might complete a test for homework, or a teacher might ask just a few students to complete a test because misconceptions are suspected among the group. The diagnostic information and teaching suggestions are available almost immediately after students complete the smart test. Results can be easily downloaded to a spreadsheet for record keeping or more detailed analysis.

Conclusion

The usability of the smart tests and the opportunities they provide for quick diagnosis in a specific topic can be very helpful for busy teachers. However the task is not yet complete. In the future we would like to increase the range of tests available to Year 5 and 6 students; to refine the power of the diagnostic algorithms; to extend the teaching suggestions; and to make it possible for all tests to be used on tablets (most already are) as well as desktop and notebook computers.

We know that the smart test system can be a powerful resource for diagnosing students' thinking, easy for schools to use, informative for teachers, and thus an important component of the assessment for learning process, and we hope that they prove so for you too. We look forward to working with any interested teachers and schools.

Beth Price, Kaye Stacey, Vicki Steinle, and Eugene Gvozdenko, The University of Melbourne

Acknowledgements

This article is an updated version of Price, Stacey, Steinle, Chick, Gvozdenko (2009), and includes new examples. The project was initially supported by Australian Research Council Linkage Projects grant LP0882176 in partnership with the Department of Education and Early Childhood Development, Victoria. Further funding has also come from the Catholic Education Office (Melbourne).

References to be found on page 26.

Having found formulas for the areas of my squares and graphed them against each other to see which was minimum when, I realised that I could answer the question as easily by considering the graphs of the sides of the squares, which were mainly linear and much easier to interpret.

And having found that as the base angle increases from 45° triangle **b** is at first smaller than triangle **e** but at some point they are equal and then triangle **e** becomes smaller, it took me a long time to find the exact value where they are equal, and convince myself why this had to be true, at which point it became totally obvious. I'll leave that for you to think about.

For me this whole wonderful investigation, which still has more for me to think about, illustrates how messy and exciting the process of eventually finding a simple, or elegant, solution to a problem can be. I worry that many mathematics books present simple solutions to students, which help the students to follow logical arguments and see relationships, but make it hard for them to understand how they could ever arrive at such ideas for themselves, or realise that most people who have reached elegant explanations or proofs have struggled with messier and more complex approaches first.

It seems important that mathematics educators find ways to help students to value and enjoy such processes, rather than feeling a weighty sense that they 'should' have seen the simple option immediately.

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Working with Laura Stockley, Maria Pittaway and Joanne Hirst on this problems and others, with many thanks for their many valuable insights on their own mathematical processes and those of their students.

Note: I have various Geogebra, Excel and Autograph files that I have used in working on this problem, and my summary of my final solution.

These can be found at www.atm.org.uk/mt235

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